ON QUANTUM MECHANICS IN A CURVED SPACETIME WITH ABSOLUTE TIME

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Abstract

We present a new covariant approach to the quantum mechanics of a charged 1/2-spin particle in given electromagnetic and gravitational fields. The background space is assumed to be a curved Galileian spacetime, that is a curved space-time with absolute time. This setting is intended both as a suitable approximation for the case of low speeds and feeble gravitational fields, and as a guide for eventual extension to fully Einsteinian spacetime. Moreover, in the flat spacetime case one completely recovers standard non-relativistic quantum mechanics.

1. INTRODUCTION

Recently Jadczyk and Modugno[1,2] have proposed a new covariant formulation of the quantum mechanics of scalar charged particles interacting with given classical gravitational and electromagnetic fields, in the framework of a general relativistic Galileian spacetime. In this paper we extend this formulation to the quantum mechanics of a particle with spin.

Our work is related to a wide literature on the classical and quantum Galilei theory starting from E. Cartan (several quotations on this subject may be found in the above papers by Jadczyk and Modugno; in particular we recall the works by C. Duval, H. P. Künzle, P. Havas, K. Kuchař, J. Ehlers, J. M. Lévy-Leblond, M. Mangiarotti, M. Modugno, W.Pauli, E. Prugovečki, E. Schmutzer and J. Plebanski, A. Trautman, W. M. Tulczyjew). Moreover, our theory has evident relations but also important differences with the geometric quantisation. We stress that the touchstone of our approach is standard quantum mechanics.

The scheme of our approach is briefly the following.

First, we sketch the essential features of our background classical spacetime. We assume a 4-dimensional spacetime fibred over time and equipped with a space-like Euclidean metric, a time-preserving linear connection (the gravitational field) and a 2-form (the electromagnetic field). We couple the gravitational and electromagnetic fields and obtain "total" geometric objects, including a cosymplectic 2-form. We postulate the closure of this form thus yielding a link between the above geometrical structures and the first Maxwell equation; moreover, we postulate a kind of "reduced" Einstein and