A FAMILY OF NONLINEAR SCHRÖDINGER EQUATIONS: LINEARIZING TRANSFORMATIONS AND RESULTING STRUCTURE

H.-D. Doebner,[1,2] G. A. Goldin[3] and P. Nattermann[2]

[1] Arnold Sommerfeld Institute for Mathematical Physics
[2] Institute for Theoretical Physics
Technical University of Clausthal
D-38678 Clausthal-Zellerfeld, Germany
[3] Departments of Mathematics and Physics
Rutgers University
New Brunswick, New Jersey 08903, USA

Abstract

We examine a recently proposed family of nonlinear Schrödinger equations with respect to a group of transformations that linearize a subfamily of them. We investigate the structure of the whole family with respect to the linearizing transformations, and propose a new, invariant parameterization.

1. INTRODUCTION

Previous work[1−5] on the representation theory of an infinite-dimensional kinematical algebra on $\mathbb{R}^3$, and the corresponding infinite-dimensional group, led to a Fokker-Planck type of equation for the quantum-mechanical probability density and current,

$$\partial_t \rho = -\vec{\nabla}\cdot\vec{j} + D\Delta\rho\,, \tag{1.1}$$

and in turn to a family $\mathcal{F}_D$ of nonlinear Schrödinger equations. $\mathcal{F}_D$ is parameterized by the classification parameter D of the unitarily inequivalent group representations (the diffusion coefficient in Eq. (1.1)), and five real model parameters $D'c_1, \ldots, D'c_5$:

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m}\Delta + V(\vec{x})\right)\psi + i\frac{\hbar D}{2}\frac{\Delta\rho}{\rho}\psi + \hbar D'\left(\sum_{j=1}^{5} c_j R_j[\psi]\right)\psi\,. \tag{1.2}$$

Here D' also has the dimensions of a diffusion coefficient (so that the c_j are dimensionless), and the nonlinear functionals R_j are complex homogeneous of degree zero, defined by:

$$R_1[\psi] := \frac{\vec{\nabla}\cdot\vec{J}}{\rho}\,, \qquad R_2[\psi] := \frac{\Delta\rho}{\rho}\,, \qquad R_3[\psi] := \frac{\vec{J}^2}{\rho^2}\,,$$
$$R_4[\psi] := \frac{\vec{J}\cdot\vec{\nabla}\rho}{\rho^2}\,, \qquad R_5[\psi] := \frac{(\vec{\nabla}\rho)^2}{\rho^2}\,, \tag{1.3}$$