DIFFEOMORPHISM GROUPS AND ANYON FIELDS

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Abstract

We make use of unitary representations of the group of diffeomorphisms of the plane to construct an explicit field theory of anyons. The resulting anyon fields satisfy q-commutators, where q is the well-known phase shift associated with a single counterclockwise exchange of a pair of anyons. Our method uses a realization of the braid group by means of paths in the plane, that transform naturally under diffeomorphisms of $\mathbf{R}^2$

1. DIFFEOMORPHISM GROUP REPRESENTATIONS AND ANYONS

The intrinsic structure of standard quantum mechanics includes representations of an infinite-dimensional group, whose infinitesimal generators are the mass density operator $\rho(\mathbf{x},t)$ and the momentum density operator $\mathbf{J}(\mathbf{x},t)$. By examining the commutation relations and other properties of ρ and $\mathbf{J}$, one determines that the corresponding group is the natural semidirect product $G = \mathcal{S}(M) \times \mathit{Diff}(M)$, where the manifold M is physical space (typically $\mathbf{R}^3$), $\mathcal{S}(M)$ is the additive group of smooth, real-valued scalar functions on M that together with all derivatives vanish rapidly at infinity (Schwartz' space), and $\mathit{Diff}(M)$ is the group of diffeomorphisms of M under composition that, together with all derivatives, become rapidly trivial at infinity.[1]

Quantum-mechanical systems are described by the continuous unitary representations (CURs) of G, or in certain cases (such as an ideal, incompressible fluid) particular subgroups of G (e.g., the volume-preserving diffeomorphisms). This fact has been established and used in our previous work to obtain a unified description of an astonishing variety of quantum systems, ranging from extended objects such as vortex configurations[2] to point particles obeying boson, fermion, and (in two space dimensions) intermediate, or "anyon", statistics.[3] The latter possibility had already been conjectured from the topology of two-particle configuration space in the plane;[4] the diffeomorphism group approach provided a rigorous prediction even without the assumed exclusion of configurations where the particle coordinates coincide. From diffeomorphism group representations there also followed many of the fundamental physical properties of anyons–the shifts in angular momentum and energy spectra, the connection with configuration space topology, the relation to charged particles circling regions of magnetic flux, and the mathematical role of the braid group.[3,5,6] Anyon statistics