DIFFERENTIAL FORMS ON THE SKYRMION BUNDLE

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1. INTRODUCTION

During the last years many physicists seem to have rediscovered the SKYRME model[1] in theoretical nuclear physics as an effective field theory related to quantum chromodynamics (QCD) by its underlying symmetry. Yet most of the articles deal with the ungauged, purely hadronic case treating interactions between baryons and mesons and do not cover interactions between these particles and electromagnetic fields, although especially for the latter case, the SKYRME model reveals some interesting features. Grand unification theory in its present form implies that magnetic monopoles M are able to catalyze baryon-number-violating processes like

$$\mathrm{M} + p^+ \longrightarrow \mathrm{M} + e^+ + \text{ pions},$$

and these processes can conveniently be described within the SKYRME model.[2]

The correct settings for the purpose of treating interactions with electromagnetic fields are that of a skyrmion bundle (and a lepton bundle) associated with a principal U_1 bundle and a MAXWELL connection on it. The possibility of describing baryonic processes by means of the mesonic fields alone, is essentially based on the topological properties of the unitary groups SU_m. While their stable homotopy groups and their DE RHAM cohomology including the generators ω_{2n+1} of $H^{2n+1}(\mathrm{SU}_m)$ are well known, little is noted on the topology of the skyrmion bundle. In order to treat baryon-number-violating processes, one needs an analog of the (normalized) differential form ω_3, which counts the number of baryons described by a certain mesonic field configuration.

Also ω_5, which serves as a base for the anomalous action, has to be generalized to the bundle case. To this end, we have examined the cohomology of the skyrmion bundle in general.[3] For the purpose of recovering (global) differential forms on the bundle from those on the fiber SU_m, we have used spectral sequences. In a second step we then gauged these differential forms by adapting them to the given MAXWELL connection.