CLASSICAL YANG-MILLS AND DIRAC FIELDS IN THE MINKOWSKI SPACE AND IN A BAG

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Abstract

Extended and reduced phase spaces for minimally interacting Yang-Mills and Dirac fields in the Minkowski space-time and in a bag are discussed.

1. INTRODUCTION

This lecture is based on joint research with Günter Schwarz devoted to the study of the canonical structure of minimally interacting Yang-Mills and Dirac fields,[1-5] and also on some earlier works.[6,7] We are going to discuss the general structure of the theory. At present we have results for Yang-Mills and Dirac fields in the Minkowski space-time, and in spatially bounded domains under the M.I.T. bag boundary conditions.

We start with the study of the existence and uniqueness theorems for the evolution equations in order to determine the extended phase space P of the system. The constraint equation is preserved by the evolution, hence the constraint set it defines is stable under the evolution.

For a given extended phase space P, the group of gauge symmetries $GS(P)$ consists of gauge transformations preserving its structure. Hence, the topology of the extended phase space determines the topology of the group of gauge symmetries.

Extended phase spaces under considerations are weakly symplectic, and the action of the group $GS(P)$ of gauge symmetries of P is Hamiltonian. The corresponding momentum map J has values in the dual $gs(P)^*$ of the Lie algebra $gs(P)$ of $GS(P)$. The constraint equation defines an ideal $gs(P)_0$ in $gs(P)$ such that the constraint set is the zero level of the restriction J_0 of the momentum map J to $gs(P)_0$. J_0 is the momentum map for the Hamiltonian action in P of the normal subgroup $GS(P)_0$ of $GS(P)$ generated by $gs(P)_0$. The reduced phase space $\check{P}$ is defined as the space of $GS(P)_0$-orbits in the constraint set ${J_0}^{-1}(0)$.

Let P_M denote the extended phase space in the Minkowski space-time theory. We find that the action of $GS(P_M)_0$ is free and proper, the constraint set ${J_0}^{-1}(0)$ is a submanifold of P_M, and the reduced phase space $\check{P}_M$ is a weakly symplectic quotient manifold of ${J_0}^{-1}(0)$ with an exact symplectic form.