COHERENT STATES : A GENERAL FORMALISM

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ABSTRACT

We present a general formalism for the construction of coherent states, based on the notion of reproducing triple. It covers the case of continuous frames in Hilbert space, as well as generalized coherent states associated to group representations which are square integrable only on a homogeneous space.

Coherent states (CS), originally introduced by Schrödinger in the context of a harmonic oscillator, later popularized by Glauber and Klauder for the description of coherent light, have been generalized to such an extent that they find applications in every single corner of quantum theory[1,2]. Yet there are cases where the known methods fail to generate CS, for instance, the Galilei or the Poincaré groups (in 1+1 or 1+3 dimensions), and other groups of the same type. Our aim here is to treat such situations, and in fact much more general ones, by a suitable extension of the notion of coherent states. The discussion is based on joint work with S.T. Ali and J.-P. Gazeau[3–5]. First we briefly review the standard method.

1. Introduction: Canonical Coherent States

Canonical CS may be viewed from two different vantage points.

(1) Starting from the usual oscillator CCR, $[a, a^+] = I$, one may define CS either as eigenvectors of the annihilation operator a, as states of minimal uncertainty (quasi-classical states) or as states obtained from the ground state by displacement operators $D(z) = \exp(za^+ - \bar{z}a), z \in C$. Thus one gets an overcomplete family of states $\{|z\rangle, z \in \mathbf{C}\}$, which are never orthogonal to each other and determine a *resolution of the identity:*

$$\int_{\mathbf{C}} |z\rangle \langle z| \, \frac{d^2 z}{\pi} = I. \tag{1.1}$$

Hence the function $K(z', z) = \langle z'|z\rangle$ is a *reproducing kernel*, which leads to the CS or Fock-Bargmann representation of the harmonic oscillator through the map $W : \psi \mapsto \psi(z) = \langle z|\psi\rangle$. Indeed W is a unitary map onto a closed subspace of $L^2(\mathbf{C}, d^2 z/\pi)$, the corresponding orthogonal projection being the integral operator with kernel K. Furthermore, the relation $z = q + ip$ allows the identification of the z-plane with the phase space of the system.

(2) Alternatively, one notices that $\{a, a^+, I\}$ is the Lie algebra of the Weyl-Heisenberg group G_{WH}. The elements of G_{WH} may be written as $(s, z) \in S^1 \times \mathbf{C}$, and