QUANTIZATION ON COADJOINT ORBITS OF DIFFEOMORPHISM GROUPS: SOME RESEARCH DIRECTIONS

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ABSTRACT

Some ongoing work is described where representations of groups of diffeomorphisms, algebras of vector fields, and their semidirect products give interesting results in quantum theory. Included are discussions of single-particle coadjoint orbits for coherent states quantization (partly with S. T. Ali), nonlinear quantum mechanics from a family of related coadjoint orbits (partly with H.-D. Doebner), and prospects for constructing measures quasi-invariant for diffeomorphisms on some new, infinite-dimensional configuration spaces (in progress with U. Moschella).

1. Introduction

In this paper I shall try to present in a fairly self-contained way some recent investigations, partially in collaboration with others, involving infinite-dimensional Lie groups and algebras in quantum mechanics–especially groups of diffeomorphisms, algebras of vector fields, and their semidirect products. The representation theory of such groups is a challenging subject. It offers a domain where ideas from the theory of coherent states quantization and geometric quantization (reviewed here in Bialowieza by J.-P. Antoine, S. T. Ali, J. Sniatycki, N. M. J. Woodhouse, and a number of others) can be extended significantly. Some of the research directions I shall discuss are not fully developed yet, but reflect work in progress with several different colleagues. Related results are summarized in the references[1].

1.1. An Infinite-Dimensional Lie Algebra in Quantum Mechanics

The idea of basing a general framework for nonrelativistic quantum theory on representations of an infinite-dimensional Lie algebra of local densities and currents was introduced and developed nearly a quarter century ago.

In 1968, R. Dashen and D. H. Sharp proposed to describe quantum mechanics by means of the formal, singular, "fixed-time" algebra satisfied by the mass density $\rho(\mathbf{x}) = m\psi^*(\mathbf{x})\psi(\mathbf{x})$ and momentum density $\mathbf{J}(\mathbf{x}) = (\hbar/2i)\,[\psi^*(\mathbf{x})\,\nabla\psi(\mathbf{x}) - (\nabla\psi^*(\mathbf{x}))\psi(\mathbf{x})]$, where ψ and ψ^* are second-quantized fields satisfying canonical commutation or anticommutation relations. Shortly thereafter, Sharp and I found we could interpret $\rho(\mathbf{x})$ and $\mathbf{J}(\mathbf{x})$ rigorously as operator-valued distributions, modeled on spaces of C^∞ scalar functions and vector fields on $\mathbf{R}^3$ that vanish (rapidly, with all derivatives) at infinity[2]. The result is a commutator algebra of gauge-invariant observables (i.e., a "local current algebra"), generated by self-adjoint operators $\rho(f)$ and $J(\mathbf{g})$ that are indexed by the functions f and vector fields $\mathbf{g}$. One writes $\rho(f) = \int \rho(\mathbf{x}) f(\mathbf{x}) d\mathbf{x}$, and