AN APPLICATION OF GEOMETRIC QUANTIZATION AND COHERENT STATES TO VORTEX THEORY

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ABSTRACT

In this note we describe some recent applications of symplectic methods, geometric quantization, and coherent states to fluid mechanics.

1. Introduction

In this note we report on recent work devoted to some applications of symplectic methods, geometric quantization, and coherent states to fluid mechanics.

We deal with the symplectic approach to classical and quantum vortices developed in [16], [17], building on the Marsden-Weinstein theory ([12]). This kind of approach has been also independently, and differently pursued by Goldin, Menikoff and Sharp, and by Ali and Goldin ([6], [7], [2], and these proceedings). There are also related studies of Rosensteel (cf. these proceedings) on the dynamics of stars and galaxies involving the same kind of notions.

The rest of this section closely follows part of the exposition given in [18] and outlines the basic results of [17]. More details will be provided in the following sections, which basically summarize the exposition given in [17].

The natural phase space of a classical vortex system is a coadjoint orbit $O_{\mathbf{w}}$ of the group $G := s\ \mathrm{Diff}\,(\mathbb{R}^3)$ consisting of measure preserving diffeomorphisms of $\mathbb{R}^3$ (rapidly approaching the identity at infinity) and it is labelled by the vorticity field $\mathbf{W} = \mathrm{curl}\ \mathbf{V}$, $\mathbf{V}$ being the velocity field of a classical perfect fluid ([12]; [6], [7], [16], [17]).

According to the Kirillov-Kostant-Souriau (KKS) theory (see e.g. [9], [10], [24], [8]), $O_{\mathbf{w}}$ is a hamiltonian symplectic manifold. The hamiltonian algebra associated to $O_{\mathbf{w}}$ turns out to coincide with the current algebra introduced in [21], [22]([16], [17]). The vorticity can be confined on the components of a (oriented) link. In this case $O_{\mathbf{w}}$ is formally Kählerian ([4], [16]).

If $\mathbf{W}$ is smooth, there is a natural Kähler manifold M (the Clebsch manifold) locally de-