LOOP SPACES AND A GEOMETRICAL APPROACH TO PATH INTEGRAL QUANTIZATION

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ABSTRACT

As a first step towards understanding path integrals in terms of the integration theory and differential geometry of path and loop spaces, in this article a special model is analysed: quantum mechanics on a compact Lie group G. The path integral for the propagator is shown to equal an integral over ΩG, the space of based loops on G. Geometrical features of ΩG are described, and the analogy with flag manifolds is pointed out. It is shown that the correct expression for the propagator may be obtained by the formal application of the Duistermaat-Heckman integration formula to the integral over ΩG.

1. Introduction

In recent years loop spaces have come to play an increasingly important role in the mathematical description of a wide variety of physical problems. Before surveying some of the main areas where loop spaces have appeared we will give a short explanation of what loop spaces are as mathematical objects.

The basic notion is that of a loop on a smooth manifold M, being a continuous map from the circle $\mathbf{R}/\mathbf{Z}$ to M. Depending on the type of application being considered one usually imposes additional conditions on the loop, e.g. that it be (piecewise) C^∞, or non-self-intersecting, or that it start at a preferred base point of M ("based loop"). The space of all loops of a certain type is called a loop space of M and will be denoted simply LM for the time being.

The most obvious area of mathematical physics in which loop spaces can be usefully employed is string theory, since a closed string on M is by its very nature an element of LM. We will not dwell on this particular application, however, because it is covered in the talk by Popov at this meeting.

Loop spaces have also appeared in the context of non-local variables in gauge theory and gravity. Taking as an example the theory of electromagnetism, the conventional gauge-invariant variables are $F_{\mu\nu}(x)$, i.e. the components of the electric and magnetic field, which are local in the sense that they only depend on a single space-time point x. However it has been known for a long time that one may also consider variables of the form $T[\gamma] = \exp \oint_\gamma iA$ where γ is a (piecewise smooth) loop