WEI AND NORMAN'S FORMALISM AND INVARIANTS

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1. Introduction

An operator $I(t)$ is said to be an invariant with respect to a time-dependent Hamiltonian $H(t)$ if it satisfies the equation:

$$\frac{\partial I}{\partial t} + \frac{1}{i}[I, H(t)] = 0, \tag{1}$$

where units $h = 1$ have been adopted. such definition is equivalent to say that all the matrix elements of I are invariant under the Schrödinger evolution. Eq.(1) is well known in Quantum Mechanics. Aside from a sign, it applies to any Heisenberg operator. Moreover, the same equation determines the time evolution of the density matrix $\rho(t)$ for a system whose Hamiltonian is $H(t)$. This last feature could perhaps explain why the interesting role of invariant operators for a description of pure quantum states was neglected for many years. It is a merit of Lewis and Riesenfeld (LR) to have shown that hermitean operators that satisfy Eq.(1) may be used for finding exact solutions of the Schrödinger equation[1]. Their method has the advantage of giving to the state vector of a quantum system an expansion with coefficients which are constant in time. On the other hand, this task can be accomplished only if one knows an invariant which belongs to the complete set of commuting observables of the system and in addition an orthonormal basis of eigenstates of this operator.

To reduce this problem to a manageable form, in most investigations it is assumed that the Hamiltonian $H(t)$ can be written in the form:

$$H(t) = a^i(t)H_i, \tag{2}$$

where the $a^t(t)$ are complex functions of time, while the time-independent operators H_i are basis elements of an n-dimensional Lie algebra, i.e.$[H_i, H_j] = C_{ij}^k H_k$, the C_{ij}^k being the structure constants of L. Here and in the following the latin indices will take values $1, \ldots, n$ and summation convention over repeated indices will be used.

In this paper it will be shown that for Hamiltonians described by Eq.(2) the invariants formalism provides remarkable simplifications in the procedures so far adopted for solving the Schrödinger equation. The approach followed here rests on a general representation of an invariant $I(t)$, valid for every Hamiltonian, and on the Wei and Norman's (WN) formalism[2] that was conceived just for dealing with Lie Hamiltonians like (2). Our main conclusion will be that the Schrödinger