GHOSTBUSTER'S APPROACH TO CONSTRAINTS

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ABSTRACT

The aim of this lecture is to exorcise the ghosts from geometric quantization of systems with constraints. A brief review of geometric quantization theory is given, and it is shown, that for the momentum map constraints, one can obtain a consistent theory free of ghosts.

1. Poisson Algebra

The starting point for the geometric quantization of classical systems is the Hamiltonian formulation of their dynamics. The scene of the action is the phase space P of the system endowed with a symplectic form ω, that is a closed non-degenerate 2-form on P. For finite dimensional manifolds non-degeneracy of ω implies that the left contraction with ω, is an isomorphism

$$TP \to T^*P : u \mapsto u \rfloor \omega \, .$$

For every smooth function f on P, there exists a unique vector field ξ_f on P such that

$$\xi_f \rfloor \omega = -df \, ,$$

called the Hamiltonian vector field of f. Hamiltonian vector fields preserve ω,

$$\mathcal{L}_{\xi_f} \omega = \xi_f \rfloor d\omega + d(\xi_f \rfloor \omega) = -d^2 f = 0 \, .$$

Conversely, vector fields preserving ω are locally Hamiltonian. Motions of the system with Hamiltonian H are given by the integral curves of ξ_H.

The mapping $f \mapsto \xi_f$ is a linear homomorphism of the space $C^\infty(P)$ of smooth functions into the space $\mathcal{X}(P)$ of smooth vector fields on P. Its kernel consists of locally constant functions on P. The Lie bracket in $\mathcal{X}(P)$ induces a lie bracket in $C^\infty(P)$ as follows. For f_1 and f_2 in $C^\infty(P)$,

$$[\xi_{f_1}, \xi_{f_2}] \rfloor \omega = \mathcal{L}_{\xi_{f_1}} (\xi_{f_2} \rfloor \omega) - \xi_{f_2} \rfloor \mathcal{L}_{\xi_{f_1}} \omega =$$