A COHERENT STATE ASSOCIATED WITH SHAPE-INVARIANT POTENTIALS

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Abstract

An algebraic treatment of shape-invariant potentials is discussed. By introducing an operator which reparametrizes wave functions, the shape-invariance condition can be related to a generalized Heisenberg-Weyl algebra. It is shown that this makes it possible to define a coherent state associated with the shape-invariant potentials.

1. INTRODUCTION

Coherent states play an important role in physics today.[1] The original coherent state is closely related to Heisenberg-Weyl group and this property has been extended to a number of Lie groups, and the term coherent states is now applied to many objects. Recently, a "coherent state" is proposed[2] from a quite different point of view. It is based on the idea of the reparametrization invariance property of exactly solved potentials, called shape-invariance.[3–6] This property is introduced by the use of supersymmetry in quantum mechanics.[7] The key idea of constructing a coherent state is to introduce an operator denoting the reparametrization, which makes it possible to express the shape-invariance condition as a commutation relation. In this note, we construct a coherent state associated with shape-invariant potentials based on this commutation relation.

2. SHAPE-INVARIANCE REVISITED

Let us first consider the following operators

$$D_n \equiv \frac{1}{\sqrt{2}}\left(W_n(x) + \frac{d}{dx}\right) \tag{2.1}$$

with the property

$$D_n D_n^\dagger = D_{n+1}^\dagger D_{n+1} + R_{n+1} \quad (n = 0, 1, 2, \cdots), \tag{2.2}$$

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