NONLINEAR QUANTUM MECHANICS, THE SEPARATION PROPERTY, AND A STOCHASTIC ALTERNATIVE TO CERTAIN NONLINEAR SCHRÖDINGER EQUATIONS

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Abstract

First we consider the separation property for hierarchies of nonlinear Schrödinger equations describing many-particle systems. This states that product wave functions, in the absence of interactions, evolve by the separate evolution of each factor. We show that it is compatible with the separation property to introduce, at any particular "threshold" particle number, truly new physical effects that are absent in systems of fewer particles. For example if single quantum particles satisfy the usual (linear) Schrödinger equation, a system of two particles can evolve by means of a fairly simple nonlinear Schrödinger equation. Second, in considering alternatives to nonlinearity, we observe that the considerations proposed by Doebner and Goldin that led them to a certain family of nonlinear Schrödinger equations can also be fulfilled by an interesting stochastic equation. The latter could avoid some of the fundamental difficulties that nonlinearity introduces into quantum mechanics, and should be considered seriously in its own right.

1. INTRODUCTION

The present paper, partially describing our joint work,[1] is based on the 1993 talk in Bialowieza, Poland by G. A. Goldin. We are grateful to the conference organizers for the opportunity to present our results. G. Svetlichny also thanks the Department of Mathematics, Rutgers University (New Brunswick) for hospitality during 1992-93, when this work was substantially completed.

Nonlinear Schrödinger equations are usually introduced either to describe particular physical effects phenomenologically, or to explore fundamental arguments that the basic equations of quantum mechanics might be nonlinear (with the usual linear theory only an approximation). Many different nonlinearities have been proposed;[2,3,4] those discussed here are just a small subset. In this article we explore one desirable property of N-particle hierarchies of nonlinear Schrödinger equations, the quantum-mechanical "separation property".[2] Our results contribute to a systematization through the characterization of hierarchies by means of two new quantum numbers.

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