QUANTIZATION BY MEMBRANES AND INTEGRAL REPRESENTATIONS OF WAVE FUNCTIONS

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Abstract

It is noted that a quantization satisfying Dirac axioms can be constructed in a very simple way over the space of functions on a lagrangian submanifold. Inner product in this space is described purely geometrically by integration of symplectic and curvature forms over certain membranes. A new definition of a "coherent state" is suggested and the problem of minimization of such "coherent frame" is discussed in symplest examples.

0. INTRODUCTION

In this work an approach to quantization, proposed in Refs.1–4, is discussed and generalized. The main idea is to look for solutions u of quantum equations in the following form

$$u = \int_{\Lambda} \varphi(x) u_x \, d\sigma(x). \tag{0.1}$$

Here u_x is a family of quantum states (a frame in a certain Hilbert space) parametrized by points x from a submanifold Λ in the classical phase space of the system, $d\sigma$ is a measure on this submanifold, and φ is a new wave function (certain function on Λ). Let f be the classical Hamiltonian, and $\mathbf{F}$ be the quantum Hamiltonian of the initial system; then we look for a new Hamiltonian $\hat{f}$ in the space of functions on Λ such that

$$\mathbf{F}u = \int_{\Lambda} (\hat{f}\varphi)(x) u_x \, d\sigma(x). \tag{0.2}$$

If we choose Λ, σ and the family u_x in an appropriate way, then the new representation $\hat{f}$ of the quantum system will be simpler and more convenient for investigations than the initial one.

The correspondence $f \to \hat{f}$ between classical observables (functions) on the phase space and operators on Λ is a "quantization".

Quantization and Infinite-Dimensional Systems
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