NONCOMMUTATIVE GEOMETRY AND SECOND QUANTIZATION

Władysław Marcinek

Institute of Theoretical Physics
University of Wrocław
Poland

Abstract

A particle system equipped with an arbitrary braid statistics is considered. The noncommutative differential calculus is used in order to describe the second quantization of a given system.

1. INTRODUCTION

The present paper continues our study of the noncommutative geometry and commutation relations corresponding to an arbitrary braiding in an algebraic way in analogy with the commutative case.[1-8] It is well known that the classical differential geometry can be described in terms of the algebra of smooth functions on a given manifold. The vector fields are derivations of the algebra of functions. We generalize the commutative differential geometry by replacing the algebra of functions by the braided commutative one. Let us consider this replacing in more detail. Let $\mathcal{A} := C(M)$ be an algebra of smooth functions on a manifold M. We denote by $m : \mathcal{A} \otimes \mathcal{A} \longrightarrow \mathcal{A}$ the multiplication of functions

$$m(f \otimes g)(x) := f(x)g(x)$$

for $f, g \in \mathcal{A}$. Let $T : \mathcal{A} \otimes \mathcal{A} \longrightarrow \mathcal{A} \otimes \mathcal{A}$ be the transposition operator

$$T(f \otimes g) := g \otimes f. \tag{1.1}$$

Define an evaluation mapping $\alpha : End(\mathcal{A}) \otimes \mathcal{A} \longrightarrow \mathcal{A}$ by the following formula

$$\alpha(X \otimes f) := Xf, \tag{1.2}$$

for every $X \in End(\mathcal{A})$, and $f \in \mathcal{A}$. An endomorphism $X \in End(\mathcal{A})$ such that we have the following Leibnitz rule

$$X(fg) = (Xf)g + fXg \tag{1.3}$$

is said to be a derivation of the algebra $\mathcal{A}$. The space of all derivations is denoted by $Der(\mathcal{A})$. It is easy to see that we have the following relation on the space $End(\mathcal{A}) \otimes \mathcal{A} \otimes \mathcal{A}$

$$\alpha \circ m^{(2)} = m \circ (\alpha^{(1)} + \alpha^{(2)} \circ T^{(1)}), \tag{1.4}$$

Quantization and Infinite-Dimensional Systems
Edited by J-P. Antoine *et al.*, Plenum Press, New York, 1994