PHASE COHERENT WAVELETS, FOURIER TRANSFORM, MAGNETIC RESONANCE IMAGING, AND SYNCHRONIZED TIME-DOMAIN NEURAL NETWORKS

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Abstract

Imaging and visualization in biomedical computing has rapidly emerged as a significant area of research aimed at developing approaches and tools for diagnosis of living systems. The final goal of diagnostic imaging procedures is to image the human body and its organ systems in a non-invasive way such that either tissue morphology or biomedical functional processes can be localized and quantified. The purpose of this paper is to indicate the significance of phase coherent wavelets in the field of phase and intensity preserving planar imaging and visualization. Planar imaging means the encoding of time-domain signals into two-dimensional spatial coordinate frames, whereas planar visualization is provided by the decoding procedure. Based on a phase coherent reference wave, phase coherent wavelets allow to create a link between temporal phase encoding and spatial encoding in such a way that temporal phase and spatial position in the image plane form essentially synonymous concepts which can be decoded by a two-dimensional Fourier transform. This linkage which represents a fundamental principle of quantum holography and phase coherent radargrammetric imaging of remote sensing, is the key to the implementation of synchronized time-domain neural network models and the quantum holographic technique of effective time reversal by the quantum coherent phenomenon of non-linear phase conjugation refocusing. The link is mathematically implemented by the unitary dual of the real Heisenberg nilpotent Lie group, the planar coadjoint orbits $\mathcal{O}_\nu(\nu \neq 0)$ of which being the basis of geometric quantization theory and coherent signal geometry. Fast imaging procedures need the transition to the compact Heisenberg nilmanifold which forms the quotient of the real Heisenberg nilpotent Lie group modulo its discrete Heisenberg subgroup. The principles of planar imaging using phase coherent wavelets are explained by the example of pulse Fourier transform magnetic resonance imaging (FT-MRI). Magnetic spin echo holograms form the symplectically invariant Weyl symbols of phase holograms in the selectively excited planar coadjoint orbit $\mathcal{O}_\nu$ localizing the on resonance spin isochromats with respect to a controlled magnetic field linear gradient frame. Read out visualization of the magnetic spin echo holograms is performed by a symplectic Fourier transform.