Symmetries in Mechanics: From Field Theories to Master Responses in the Constitutive Modeling of Materials

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Abstract This chapter is concerned with the role of continuous symmetries in field theories in a broad sense and in particular in continuum mechanics. In the first part, we present in a synthetic and self-contained manner the formalism of classical and quantum field theory, focusing on the essential role of symmetries in connection with the Lagrangian and Hamitonian formalism. The second part highlights the importance of symmetries in continuum mechanics and mechanics of materials. Especially, a novel rational methology for constructing constitutive models of viscous materials combining Lie symmetries with experimental data is presented.

1 Introduction

This chapter is concerned with the role of continuous symmetries in field theories in a broad sense and in particular in continuum mechanics. In classical mechanics, interactions between particles are supposed to occur instantaneously, and the space-time, conceived as the frame of evolution of all physical phenomena, is Newtonian, so that the time is absolute. Those interactions are described by adding to the Lagrangian, only function of the particles position (and not of velocity), a supplementary term coined interaction potential. The picture is completely different in field theory, due to the finite speed of propagation of interactions evidenced by Michelson and Morley experiment, leading to special relativity: the forces acting at a given moment on any particle are not defined by the position of these particles at the same instant. *A* finite duration of the propagation is thus required so that the change of position of a given particle produces some effect on the other particles. In order to account for this modification, one introduces the concept of field: to the idea of an action of a

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